Session Goals:
- To identify the five mathematical processes and how these can be implemented into the K-1 classroom to enhance teaching and student learning.
- To explore teaching episodes using the process standards and mathematical ideas as lenses to reflect upon teaching, learning, and mathematics.
- To consider how different strategies can be used to solve the same task.

Session Objectives:
- Participants will be able to identify the process standards embedded in teaching episodes.
- Participants will be able to explain how the process standards impact students’ learning.
- The participants will be able to use different mental math strategies (i.e., with their heads, does not have to be done in their heads) to solve addition and subtraction problems.

Corresponding SOLs:
- Process standards in the narrative prior to the listing of the Standards of Learning.
- The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts. (While we do not address this SOL specifically, we are addressing the idea of the kinds of mathematical thinking that need to take place prior to students being asked to recall basic facts.)

Class Activities:

1. **Introductions, collection of student work from assigned problem** (no more than 10 minutes)
   - Ask teachers to introduce themselves by sharing their name, school, grade level, and most interesting fact about themselves.
   - Ask for volunteers to share insights they learned through completing the student assessment.

2. **Course syllabus** (no more than 5 minutes)
   - Before handing out the syllabus, begin by telling the participants that one of the main ideas in this course is about developing and enhancing number sense. Ask participants to write on index cards what number sense means to them (it can be narrative or bulleted ideas). Then ask for volunteers to share their ideas.
Three definitions of number sense are:

• “good intuition about numbers and their relationships.” It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (Howden, 1989).

• Flexibility in thinking about numbers and their relationships.

• “Two hallmarks of number sense are flexible strategy use and the ability to look at a computation problem and play with the numbers to solve with an efficient strategy” (Cameron, Hersch, Fosnot, 2004, p. 5).

3. Pre-assessment (no more than 30 minutes)
Explain to participants that this is a necessary part of the professional development. We need to know if what we are doing is making a difference so they will be asked to complete a post-assessment at the end of the professional development as well. Try to assure participants that the pre- and post-assessments are not punitive and will be used to inform what we are doing. Encourage them to do their best.

4. Process Standards (no more than 30 minutes)
Introduce the National Council of Teachers of Mathematics Process Standards from the Principles and Standards of School Mathematics (PSSM) (2000) using the following activity. Remind teachers that the process standards are included in the narrative before the mathematics SOLs. Explain that the purpose of this activity is to help them develop or articulate a working definition of each of the process standards to help guide teaching mathematics.

Group teachers into groups of 2-3 and give them the names of the process standards (problem solving, reasoning and proof, communication, connections, representation) and ask them to describe the essence of each of the process standards either using a couple of sentences or bulleted ideas. Tell them these descriptions do not have to be all-encompassing, but just the ideas that come to mind when they think of these processes. Take about 15-20 minutes of the allotted time to facilitate a discussion using the teachers’ ideas as well as ideas from the NCTM process standards (See Handout 1.1 Mathematics Process Standards). You may wish to capture teachers’ ideas on poster sized post-it notes as a form of formative assessment regarding their perceptions of the process standards.

Share Handout 1.2 (The Use of Questioning) as an example of how a teacher can use questions to enhance the communication in a classroom that emphasizes reasoning and making sense of the mathematical ideas being learned.
More information pertaining to PSSM can be found at http://www.nctm.org/standards/content.aspx?id=16909. If facilitators and teachers are not members of NCTM, they can sign up for a free trial and gain full access to the online version of PSSM for 120 days. Facilitators may want to encourage teachers to join so they can access all of the online resources NCTM offers. Encourage them to join, even if they wait until after the class ends. NCTM offers so much in the form of professional development and great resources that most elementary teachers are not even aware of.

5. Doing mathematics. (About 30 minutes)

"How can teachers teach a mathematics that they never have learned, in ways that they never experienced?"

- From Policy and Practice: An Overview, by David Cohen and Deborah Ball

In order to understand how to teach mathematics in ways that are different from a teacher-centered, lecture style format (i.e., I tell, you do), teachers need to engage with mathematical tasks in ways that emphasize the process standards and specifically with making sense of the mathematics. Throughout the module we will pose problems for teachers to solve. They will share their ideas during a discussion period in class, through posting their ideas to an online discussion board, or submitting a written document to the instructor.

When posing problems in class, provide appropriate manipulatives and other tools that you think might help support their problem solving. Ask them to work on their own before they start working with partners or in groups of 3. As teachers work, ask questions about any pictures they are drawing, numbers they are using, strategies they are trying.

After groups have had some time to work on the problem ask particular individuals to share some of their ideas with the whole group. Try to purposefully choose individuals who have used a drawing, numbers, strategies, or something that you wish to highlight to the rest of the group. Accept teachers’ responses without evaluating them, asking others to comment on each others’ responses and insights. When possible, ask probing questions to get teachers to further explain what they mean and how their ideas might be connected to something else that has been shared (e.g., other teachers’ comments, the process standard discussion, etc.) – even if you already know the answers to these questions – others in the class may not be tracking on these ideas, so you need to attempt to focus their attention towards these ideas. Remember you are beginning to build a community of learners and building trust in each other is part of this community building. Participants need to know their ideas will be accepted at some level, so look for
opportunities to do so. Look for participants who are not readily participating and sharing their ideas and make sure to interact with them at least on an individual basis to increase their confidence and their trust.

We start with the following problems because they should be familiar enough to participants that they are willing to take risks in solving them in different ways. You will want to tell them to solve the problems as adults, not as they would expect a student in K-2 grades to solve them. Otherwise some participants will draw pictures and count by ones, mimicking how they would expect young children to solve the problems. Remind them these are problems for them to work through in order to better develop and refine their own mathematical understanding.

Problem 1: Ask participants to put down all pencils and pens and tell them you want them to do the following problem mentally. Cover up the task until you are ready for them to work on it.

\[
\begin{align*}
25 \\
+ 98
\end{align*}
\]

First ask for the answer (123) and verify that everyone agrees that the answer is 123. Then ask for volunteers to share how they figured out the answer. You will have some say they did the standard procedure in their head. Ask them to state what they thought about. Ask how many participants did it this way. Then ask if anyone figured out the answer in a different way. If no one used a different method, ask them to work in pairs to see if they can identify a different way, paying attention to the numbers themselves. Typically, someone has seen that 98 is close to 100 and so they use this idea to either add 100 to 25 and then subtract 2 to get 123 or they take 2 from the 25 to get 98 to 100 and then add 23 to 100. Continue to ask if anyone used a different method or now sees a different method. Another way that could be used is to add the tens first to get 20 + 90 or 110 and then add the ones, 5 to 110 to get 115 and then 8 more to get 123. This last step could be found by splitting 8 into a 5 and 3 to get from 115 to 120 and then using the 3 to get to 123. If no one suggests these ideas, share them as ways you have seen others use. As participants share different methods, have them record their ideas on the board or you record their ideas on the board. After you have captured various ways of thinking about adding these two numbers, ask the participants to look back over the different methods and identify the mathematical ideas involved in the different strategies. The table below shows some of the various methods and the corresponding mathematical ideas.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mathematical Ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Procedure</td>
<td>Value of digits; sums of single digit numbers</td>
</tr>
<tr>
<td>Used the idea that 98 was close to 100.</td>
<td>Number relationships – knew 98 was close</td>
</tr>
</tbody>
</table>
Either added 100 and 25 and subtracted 2 or used compensation by taking 2 from 25 to get 98 to 100 and then adding 23.

- to 100; knew 23 was 2 less than 25.
- Equivalence- knew adding 23 to 100 was the same as adding 25 to 98; knew that in order to maintain the total, needed to subtract 2 when added 100 to 25.

<table>
<thead>
<tr>
<th>Adding tens and then adding ones.</th>
<th>Place value – knew the 9 in 98 was 90 and the 2 in 23 was 20.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number relationships – when splitting the 8 into a 5 and a 3, knew that 115 was 5 away from 120, so looked for a way to get 5 from 8.</td>
</tr>
</tbody>
</table>

### Problem 2:

Ask participants to solve these two problems: 272 – 14 and 283 – 275 (not as they think elementary school students would, but as adults might).

After they have had some time to solve the problems, ask participants to write a response to the following questions: Did you use the same strategy for both problems? Why or why not?

Ask for volunteers to share how they thought about the problems. If everyone used the standard subtraction procedure, ask them to try to solve one or both of these problems using some kind of mental math strategy (i.e., based on number sense; with your head, does not have to be done in your head) that does not rely on the standard procedure.

Ask volunteers for their ideas. If necessary, share the following strategies with them, using an open number line to record the jumps:

- For 272 – 14: Go back 10 from 272 to 262. Then split the 4 into 2 and 2 and go back 2 from 262 to 260 and then again to 258. The answer is 258.
- For 283 – 275: Add 5 to 275 to get to 280. Then add 3 more to get to 283. The answer is 8.

Note that some participants will not agree that the second bullet represents subtraction because you used addition to find the answer. Facilitate a discussion about what is subtraction. Many participants will think of subtraction as only used in “take away” situations. A broader definition of subtraction is comparing or finding the difference between two numbers. Subtraction is more than just “take away.”

Facilitate a discussion about why it might make sense to use different strategies for these two tasks. One of the reasons we teach standard computational procedures is for efficiency but oftentimes we do this at the expense of
developing number sense. Which is a more efficient strategy for the task 283 – 75, the standard algorithm or adding up? The idea of “looking at the numbers before choosing a strategy” is likely to be a new concept to participants. Encourage participants to consider the potential for students to develop number sense with this new concept.

Reflection Time: After participants have engaged in these tasks, ask them to discuss in groups of 2-3 where we used the process standards and how it impacted their learning. Encourage them to reflect upon how this experience was different/similar to how they had engaged with mathematics in the past. The points you are looking to highlight are that they were not told they had to use a particular strategy to complete the task. They could use whatever ideas made sense to them. They were also asked to explain how they reasoned through the tasks, which could have benefitted them by helping them clarify their ideas or benefitted others by highlighting new ideas and strategies. They should also note that many different ways were accepted as valid ways to complete the task. Point out if different representations were used and how those could help to make particular mathematical ideas or connections more explicit.

5. Video of kindergarten classroom and discussion (Cognitively Guided Instruction blue CD) (no more than 15 minutes; video lasts 7 minutes)
Before showing the video of the kindergarten classroom inform the teachers that the children they are getting ready to watch are not a gifted group of children but are the most advanced students in this kindergarten classroom. It is later in the year and the children have been in a classroom which readily emphasizes the process standards. The teacher also uses research-based frameworks to inform planning, instruction, and assessment – frameworks they will be learning about during this professional development experience. Provide the following prompts for teachers to guide their watching of the video:
   a. What are the mathematical ideas children are using and exploring?
   b. Where are the process standards at play in this classroom episode and how do those seem to affect students’ learning?
   c. Notice how the teacher interacts with the children. What kinds of statements and questions does he use?
   d. What about the video strikes you as significant?

After watching the video, facilitate a discussion using the above questions as a guide. Remember to look for ways to build a community of learners (see text in #4 above).

6. Introduction to Young Mathematicians at Work (YMW) (no more than 5 minutes)
You may wish to spend some time introducing some of the terms in the first three chapters of the YMW book. In particular, subitizing and landmarks are terms you may wish to introduce to teachers prior to their beginning the book. Subitizing is a
Strategy for being able to know how many objects are in a small set without having to count. Studies have shown that even babies can distinguish objects up to 3. Children can usually subitize quantities of five or less. Consequently, this information can be used to help structure tasks to help students develop more efficient ways of operating on number. You may wish to show participants Figure 9.6 on p. 165 as an illustration of the landscape of learning.

7. Go over Homework, including introduction to the online environment where participants will post online discussion responses. (about 20 minutes)

Homework:

1. a. Read Chapters 1-3 of Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction. (YMW)

   b. Participate in an online discussion with your colleagues by posting your responses to the discussion questions (see below) and then post meaningful comments to at least one colleague’s postings by the dates provided by your instructor. (“I agree” is not a meaningful comment. You can agree, but you have to explicitly state why you agree or don’t agree with someone’s posting.)

   Discussion Questions:
   1. In Chapter 2 (p. 16), the authors state, “One important finding is that children do not all think the same way.” This idea is also further developed in Chapter 3. How does this statement support or challenge your notions of teaching mathematics?
   2. What questions has this reading prompted for you?

   c. Come to class prepared to discuss the following:
   • The differences and relationships between landmarks, big ideas, strategies, and models.
   • Where you see the process standards at play in Chapters 1-3 of YMW.

2. Mathematical problem. Your task is to find two different ways to solve the following problem that do not use drawings of single objects or objects grouped into groups of ten or the standard procedure for subtraction. In other words, use number relationships to find different ways to solve the problem.

   $93 - 69 = ?$
You may use some kind of manipulative or representation (like the open number line introduced in class) as you think about this task. Write up your ideas and send to your instructor by the date provided by your instructor. Your ideas should include drawings (Microsoft Word has drawing tools), numbers, strategies you used to think about the problem.

References


